A Novel Investigation of Rotating and Stratified Flows

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The global ocean circulation current (thermohaline circulation) is a slow, large scale feature which originates in a relatively fast, small scale phenomenon known as the deep-ocean overflow (see Figure 1). The coupling of the dynamics of the small fast motions to the large slow motions is correspondingly a function of the degree to which the various scales experience the rotation of the earth and the stratification of the ocean due to gravity. Motivated by the need to understand this type of scale-linking, we have obtained new mathematical results that describe the statistical behavior of rotating and stratified flows as a function of the relevant parameters, and hence as a function of scale.

Instead of looking at the dynamics of energy, the most common approach in much of fluid dynamics research, we take advantage of a quantity known as the *potential vorticity* and its square the *potential enstrophy*. We take such an approach because the distribution of total energy in space is independent of the parameters governing the rotation rate (Rossby number Ro) and the degree of stratification (Froude number Fr). Potential vorticity and potential enstrophy, on the other hand, retain dependence on Ro and Fr at essentially all scales and therefore provide a better handle on the parameter-dependence of the dynamics. Our results are a first step toward a complete understanding of the parameter-dependencies of rotating and stratified flows; these results will have significant impact on modeling such flows over a wide range of parameters and scales.

The dynamics of the oceans and the atmosphere are constrained by two important physical phenomena: the rotation of the earth, and the

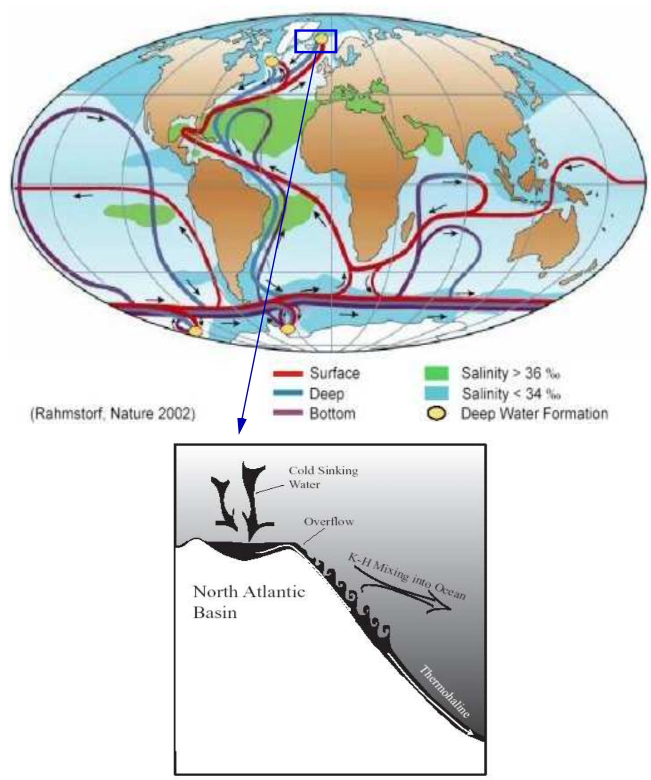

Figure 1: The global ocean circulation current is many thousands of miles in scale and has a thousand-year cycle (top). But it is driven by a small-scale, fast phenomenon, a few hundred meters across: the continuous overflow of dense cold water in the deep ocean down to the ocean floor (bottom).

downward pull of gravity. Rotation is partly responsible for characteristic features such as hurricanes, the north Atlantic gulf stream and the north American jet stream. The force of gravity causes stratification: heavy fluid (cold air or water) settles below lighter fluid (warm air or water). The strength of both rotation and stratification effects depends on scale. Large-scale phenomena such as hurricanes are strongly influenced by the earth's rotation, which is why hurricanes are always cyclonic (that is, rotate counter-clockwise in the northern hemisphere). On the other hand, there are small-scale motions, such as might be felt by a fish, for which the earth's rotation is a minor effect. An analogous scale-dependence holds for stratification.

In ideal rotating and stratified flows, the total vorticity (the local tendency of the flow to 'circulate') and the gradient of the density (the rate at which the density increases or decreases locally) at a point "trade-off" with each other in such a manner as to preserve their inner product, the potential vorticity $q = \boldsymbol{\omega}_a \cdot \nabla\rho$. Mathematically,

$$\frac{Dq}{Dt} = \frac{D}{Dt}(\boldsymbol{\omega}_a \cdot \nabla\rho) = 0$$

where $\frac{D}{Dt}$ denotes the total derivative, the total vorticity $\boldsymbol{\omega}_a = \nabla \times \boldsymbol{u} + 2f$, ρ is the density, $\boldsymbol{u}$ is the velocity and f is the constant frequency of the background rotation.

Potential vorticity and its square $Q = q^2$, the potential enstrophy are already well-known in the study of geophysical flows. In the limit of very high rate of rotation and strong stratification, known as the quasi-geostrophic limit, potential vorticity dynamics provide a complete description of the flow: other quantities such as wind-speed may be deduced from knowing the potential vorticity in this limit. The state-of-the-art in ocean/climate modeling assumes that the quasi-geostrophic limit, which essentially models largest scales of the flow in isolation from the small and intermediate scales, provides an adequate description. However, we know that large systems exhibit a range of spatial scales from thousands of kilometers to meters and even centimeters, with corresponding variability in Ro and Fr. The scales interact dynamically to govern ocean and atmospheric dynamics in ways that are only partially understood. For example, in certain scale regimes, intermediate-scale motions self-organize to generate larger-scale structures such as hurricanes, while in other regimes, energy is transferred from large-scale winds and tides to small-scale turbulent fluctuations. On large scales the flow becomes essentially two-dimensional, with motions mainly parallel to the surface of the earth, whereas at small scales it is often fully turbulent in all directions.

Since we need to study quantities which retain Ro and Fr-dependent information over a wide range of scales, potential vorticity and potential enstrophy are ideally suited for this investigation. Their statistical behavior in regimes other than the quasi-geostrophic have not been investigated until now. We have shown that using potential vorticity statistics, we can explore *six* different limiting cases of our equations, in the relevant non-dimensional parameters Rossby and Froude. In three of these cases, we predict *universal* statistical behavior in the range of scales much larger than those in which diffusive processes dominate. By universality we mean statistical independence from factors such as large scale boundary-conditions or external pumping (such as wind) that may be non-universal. This type of universal range of scales is called an 'inertial range', and is a benchmark observation in energy spectra in highly turbulent flows without rotation or stratification. Our results are the first to predict an inertial range of scales for potential enstrophy and furthermore, to directly address a wider space of parameters in which universality might hold.

Our analysis and its verification by simulations will improve our fundamental understanding of scale-linking in the ocean and other multiscale geophysical systems. These results will also be used to benchmark and parameterize rotating and stratified flow models just as energy dynamics are now used to study non-rotating, non-stratified flows. The analytical details of this work are given in [1].

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References

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